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Unification of Quantum Theory and Relativity

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A underlying dynamical structure for both relativity and quantum theory—"superrelativity"—has been proposed in order to overcome the well-known incompatibility between these theories. The relationship between curvature of spacetime (gravity) and curvature of the projective Hilbert space of pure quantum states is established as well.

1. INTRODUCTION. ABOUT "SUPERRELATIVITY"

A new principle of "superrelativity" (SuperR) has been discussed in previous reports (Leifer, 1996a, b). In the framework of this principle a nonlinear equation of motion for a relativistic scalar field [see (6.23) in Leifer (1996a)] was established. In this work we will study the physical meaning of this equation on the basis of an approximate solution.

A few words about general properties of our approach. Notions of *material point, event*, and *classical spacetime* in both special (SR) and general relativity (GR) are liable to lead to confusion at the quantum level. Instead of these obsolete objects we use a new set of primordial elements. Namely, they are *pure quantum state, quantum transition*, and *quantum state space*, respectively. In the framework of our model the *fundamental scalar field* is rendered in a self-interacting nonlinear field configuration—"droplet." The proper surrounding field of the droplet is a non-Abelian (relative to the transformation group of the Fourier components of the scalar field) gauge field of the connection in the complex projective Hilbert space of pure quantum states CP(N − 1). The principle of "superequivalence" *identifies this unified gauge field with the real physical fields of nonlocal elementary particles.* That is, the "superequivalence" principle establishes a parallelism

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between GR and SuperR. This parallelism means that in GR the *freely falling frame* serves for the description of motion of the material point. In SuperR a *local functional frame* connected with proper components of the scalar field configuration serves for the description of the evolution of the *quantum state in the unified surrounding gauge field of the connection in CP(N − 1).*

The equivalence principle of Einstein (1916) is based on the experimental fact that acceleration of bodies in a gravitational field is independent of the masses of these bodies. This situation is physically equivalent to the motion of the system of bodies in an accelerated frame. We cannot, of course, put the *criterion of identical acceleration* as the basis of geometrization of quantum physical fields. In quantum field theory the notion of "acceleration" is poor at best and ambiguous at worst because a quantum particle has some internal structure. Furthermore, at a deeper level there is *entanglement and even indistinguishability of "internal" and "external" degrees of freedom.* Therefore in the quantum regime we can not act *literally* as Einstein did in GR, but only in his *spirit* (Leifer, 1996a, b).

We have put at the basis of our "superequivalence" principle the fact that in *all interactions of quantum ("elementary") particles there is a conservation law of electric charge.* Then the group of isotropy of a pure quantum state $|\Psi>$ is $H = U(1)_{el} \times U(N-1)$. For ordinary Hilbert space C(N) the variations of a pure quantum state $|\Psi>$ lie in the coset $G/H = SU(N)/S[H = U(1)_{el} \times U(N-1)]$. It is clear that variations of the pure quantum state are due to some physical interaction; the effect of the interaction has the geometric structure of a coset, i.e., the structure of the complex projective Hilbert space CP(N − 1) (Kobayashi and Nomizu, 1969)

$$G/H = SU(N)/S[U(1)_{el} \times U(N-1)] = CP(N-1) \qquad (1.1)$$

This statement has a general character and does not depend on particular properties of the pure quantum state. The reason for the change of motion of a material point is the existence of a force. The reason for the change of a pure quantum state is an interaction, which may be modeled by unitary transformations from the coset (1.1). The reaction of a material point is acceleration. The reaction of a pure quantum state is the deformation of the "ellipsoid of polarization" (Leifer, 1996a, b). One-parameter transformations from the coset create the geodesic flows which are defined by the matrix $\hat{T}(\tau, g)$, (5.7) of Leifer (1996a). Therefore geodesics in CP(N − 1) play an important, but quite different role than geodesics in GR (Leifer, 1996a, b).

In the local coordinates

$$\pi^i_{(0)} = \Psi^i/\Psi^0 \qquad (1.2)$$

one can build a local functional frame for which

$$D_\sigma(\hat{P}) = \Phi^i_\sigma(\pi, P)\frac{\delta}{\delta\pi^i} + \Phi^{i*}_\sigma(\pi, P)\frac{\delta}{\delta\pi^{i*}} \tag{1.3}$$

spans the tangent Hilbert space relative to the Fubini–Study metric (Leifer, 1996a, b)

$$G_{ik^*} = 2\hbar R^2 \frac{\left(R^2 + \sum_{s=1}^{N-1} |\pi^s|^2\right)\delta_{ik} - \pi^{i*}\pi^k}{\left(R^2 + \sum_{s=1}^{N-1} |\pi^s|^2\right)^2} \tag{1.4}$$

The coefficients of these tangent vectors are defined by

$$\Phi^i_\sigma(\pi, P) = \lim_{\varepsilon\to 0} \varepsilon^{-1}\left\{\frac{[\exp(i\varepsilon P_\sigma)]^i_m \Psi^m}{[\exp(i\varepsilon P_\sigma)]^0_m \Psi^m} - \frac{\Psi^i}{\Psi^0}\right\} = \lim_{\varepsilon\to 0} \varepsilon^{-1}\{\pi^i(\varepsilon P_\sigma) - \pi^i\} \tag{1.5}$$

As a matter of fact, one even has a generalization of the main idea of Einstein in gravity. Namely, in GR we can separate the universe into two parts—gravitational field and "matter" (Einstein, 1916). In the framework of GR the gravitational field is "dissolved" in the geometry of spacetime. In SuperR all matter is "dissolved" in the geometry of the projective Hilbert space. Therefore we have a consistent approach to the problem of the divergences, since the spacetime localization has, from this point of view, a dynamical character (Leifer, 1996a, b). We can exemplify this point in QED.

The regularization procedure is effectively the procedure of a "delocalization" of a point-charged electron. We do not know, however, the mechanism for the suppression of processes of higher orders and it is very difficult to find some physically acceptable mechanism for keeping the extended electron from flying apart (Dirac, 1962). But on closer examination we will probably find that this difficulty is not a real one; in the "geometry of the deformation of the pure quantum states" CP(N − 1) there is an absolutely natural stabilizing Goldstone and Higgs mechanism (Leifer, 1996a, b) (which requires investigation in detail). It seems much better to think of "deformation" of the quantum state and then look for localizable solutions ("droplets") of some nonlinear wave equation as a model of nonlocal quantum particles than to begin with the point-charged electron.

2. SUPERRELATIVITY AND GRAVITY

One obtains a nonlinear Klein–Gordon equation (NLKG) for the effective deformation of a quantum state by requiring that the evolution should move the quantum state along a geodesic curve in CP(N − 1) (Leifer, 1996a, b). This equation is as follows:

$$\Box\Psi^* + \Box A^* + \Psi^*_{\mu\mu}\frac{\delta A_\mu}{\delta\Psi_\mu} + \Psi^*_\mu\frac{\delta A_{\mu\mu}}{\delta\Psi_\mu} + \alpha^2\left(\Psi^* + \Delta\Psi^* + \Psi^*\frac{\delta\Delta\Psi}{\Delta\Psi}\right) = 0, \quad \text{c.c.} \tag{2.1}$$

where $A_\mu = \partial\Delta\Psi/\partial x^\mu$, $A_{\mu\mu} = \partial\Delta\Psi/\partial x^\mu\partial x^\mu$, $\Psi_\mu = \partial\Psi/\partial x^\mu$, $\Psi_{\mu\mu} = \partial\Psi/\partial x^\mu\partial x^\mu$, and

$$\Delta\Psi^i = -\frac{g\Psi^0\tau^2}{\sqrt{1 + |\Psi^0|^2/R^2}}\Gamma^i_{km}\xi^k\Psi^m \tag{2.2}$$

We seek a solution for the equation in the approximate form $\Psi = \Phi + \tau\Delta\Phi$, where Φ obeys the Klein–Gordon equation. That is, we assume that the solution of the NLKG may be represented as a solution of the ordinary Klein–Gordon equation plus some extra terms arising from the geometric gauge "potential" in CP(N − 1)

$$\Gamma^i_{kl} = -2\frac{\delta^i_k\pi^{l*} + \delta^i_l\pi^{k*}}{R^2 + \sum_s^{N-1}|\pi^s|^2} \tag{2.3}$$

It is clear that we cannot hope find a (nonperturbative) soliton-like solution of the NLKG. But our aim now is to establish locally a relationship between the spacetime structure and the curvature of the projective Hilbert space. In order to do it, let us look at the equation for a scalar field in curved spacetime, i.e.,

$$(-g)^{-1/2}\partial_\mu[(-g)^{1/2}g^{\mu\nu}\partial_\nu\Phi] + m^2\Phi + \eta\rho(x)\Phi = 0 \tag{2.4}$$

where η is a coupling parameter, $g^{\mu\nu}$ is the metric tensor, and $\rho(x)$ is the scalar curvature of spacetime (see, for example, Birrell and Davies (1982)). One may think of to extra terms in (2.1) as associated with a scalar field in a Riemannian geometry as in (2.4). I have obtained the coefficients in (2.1) in a "CP(2)-approximation" up to second order in τ with the help of a program in "Mathematica." They have a very simple structure, but many terms. If one tries to identify some terms with the Fourier components of the metric tensor $g^{\mu\nu}$, then one cannot be certain that different terms in (2.1) are the correct Fourier components of the scalar curvature $\rho(x)$ appearing in (2.4). Notwithstanding this, we can think of an "effective Riemannian geometry" of the spacetime in which fluctuations could be effectively described by the phenomenological parameters m and η. The NLKG equation, as distinct from

(2.4), contains only one free parameter, the sectional curvature $1/R^2$ of the projective Hilbert space. Note that NLKG contains a term with the fine structure constant α instead of the mass of the scalar field. This is a consequence of the choice of the "classical radius" of the meson $r_0 = e^2/mc^2$ as the unit of our scale (Leifer, 1996a, b). Such a choice is useful since the inequality

$$\sqrt{\frac{\hbar G}{c^3}} < \frac{e^2}{mc^2} < \frac{\hbar}{mc} < \frac{\hbar}{\sqrt{2m(E-U)}} \tag{2.5}$$

may be rewritten as follows:

$$\frac{m}{e}\sqrt{\frac{G}{\alpha}} < 1 < \frac{1}{\alpha} < \frac{mc}{\alpha\sqrt{2m(E-U)}} \tag{2.6}$$

This shows that besides the de Broglie envelope, long-range plane waves which depend on modulation by the "external" parameters E, U, there is a wide range of "internal" oscillations. These oscillations are connected with internal degrees of freedom and should be related to the spatial destribution of a matter carrier for these degrees of freedom. The expression for the mass distribution can be obtained under the above-mentioned assumption on an "effective Riemannian geometry." We have, in accordance with the Einstein (1916) expression for g_{00},

$$g_{00} = 1 - \frac{2GM}{c^2 r} = 1 - \frac{r_S}{r} = 1 + \Omega(x, R) \tag{2.7}$$

where $\Omega(x, R)$ is the collection of terms in the decomposition of (2.1) which corresponds to the time component of the Laplacian. Then one has a formula for the spatial distribution of mass

$$M(x, R) = \frac{c^2 r}{2G}\,\Omega_0(x, R) \tag{2.8}$$

The implication of this result and some corrections to the nuclear potential of Yukawa will be discussed elsewhere.

3. DISCUSSION

The unified structure of a "deformation" of the pure quantum state gives us the possibility to investigate some general properties of quantum systems. The nontrivial metric and topology of the projective Hilbert space presumably may endow global solutions of a nonlinear wave equation with interesting physical properties. The tangent fiber bundle of the quantum state space in our model is the main tool of the unified description of matter fields in the spirit de Broglie–Schrödinger–Bohm. The connection in the projective Hilbert space is a generalization of the well-known Panchratnam connection. This connection defines a parallel transport in CP(N − 1). A comparison of "directions" in the original spacetime is reduced to the comparison of field configurations (shapes of "ellipsoid of polarization") by parallel transport in a projective Hilbert space. Spacetime structure therefore appears to arise only "effectively" and the problem of localization may be solved in a dynamical manner [as illustrated by (2.8)].

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